POLITECNICO
MILANO 1863

Department of Aerospace Science and Technology
Doctoral Program in Aerospace Engineering

NON-IDEAL STEADY SUPERSONIC FLOWS

Doctoral dissertation of:
Davide Vimercati

Supervisor:
Prof. Alberto Guardone

Co-supervisor:
Prof. Alfred Kluwick

Tutor:
Prof. Luigi Vigevano

The Chair of the Doctoral Program:
Prof. Pierangelo Masarati

January 2019 – Cycle XXXI

SUMMARY

Compressible flows in the neighbourhood of the vapour-liquid saturation curve and critical point are found in many technical applications, including hybrid rocket engines, power cycles, natural gas extraction and pharmaceutical processes. Operating at such high densities entails phenomena that have no counterpart in the dilute, ideal-gas regime which is often assumed in the theory of gasdynamics. Examples are: the increase of the speed of sound in adiabatic expansions, the increase of the Mach number across oblique shock waves and its reduction along isentropic expansions, the admissibility of rarefaction shocks, the reflection of oblique shocks as Prandtl-Meyer fans. At the basis of these gasdynamic effects is the peculiar thermodynamic behaviour of the substance which cannot be properly modelled, neither quantitatively nor qualitatively, using the ideal gas law. To emphasise their non-ideal thermodynamic nature, effects such as those mentioned above are referred to as non-ideal effects.

Progress in the field of Non-Ideal Compressible Fluid Dynamics (NICFD)—the branch of fluid mechanics devoted to the study of compressible flows whose behaviour deviates from that predicted by the ideal-gas model—will enable the improvement of existing industrial processes and machinery exploiting substances close to their critical point, as well as support the design of new ones. The contribution to NICFD given by the study documented herein is related to steady supersonic flows and in particular to those developing in confined geometries. In this context, a fundamental research question motivates the investigation: What are the physically admissible flow configurations in the non-ideal regime and how do they differ from those arising in ideal-gas flows? Also, what are the necessary conditions to observe a specific configuration, i.e. how is the flow field linked to the properties of the substance and the boundary conditions? These are key aspects in technical applications involving non-ideal supersonic flows and questions that this work will address.

To achieve the research goal, a thorough theoretical analysis of steady supersonic flows in the non-ideal gasdynamic context is performed. Efforts concentrate on three main complementary areas: flows in converging-diverging nozzles, flows around compressive/rarefactive ramps, shock reflections and interactions. These are elementary building blocks of general internal supersonic flows. A fully non-linear analysis is carried out to complement and extend previous studies relying on asymptotic expansion theory or numerics. The converging-diverging nozzle is the prototype of a variable area duct, the flow past ramps exemplifies the abrupt deflection of a supersonic stream (of course important for external flows as well), performed, e.g., though a shock wave which will be reflected by an opposing wall or interact with another incident shock. It is assumed that thermoviscous effects can be neglected and thus that the flow can be fully described by the Euler equations coupled with the Rankine-Hugoniot relations at points of jump discontinuity. On the other hand, an arbitrary equation of state of the fluid is allowed. In particular, the usual constraints of classical gasdynamics on the curvature of the isentropes in the pressure–density or pressure–specific volume plane, which ultimately determines the qualitative evolution of the flow, are dropped.

Flows in converging-diverging nozzle are studied using the quasi-one-dimensional approxima-

tion. A novel analytical approach sheds light on the connection between a general adiabatic flow field and the underlying local isentropic-flow features, including their qualitative change across shock waves. Isentropic flows are first classified to ease the construction and inspection of shocked flows, which are computed by means of shock-fitting techniques. The idea of functioning regime is introduced to analyse the response of the system to variations in the outlet pressure at fixed stagnation conditions at the nozzle inlet, i.e. fixed reservoir conditions, assuming the nozzle to be connected to an upstream reservoir. Extending previous results available in the scientific literature, ten different functioning regimes are singled out which include the ideal-gas-like scenario and nine non-ideal configurations. Key features of the non-ideal functioning regimes are the inclusion of rarefaction shocks to achieve arbitrarily large Mach numbers and the possibility of up to three shocks in nozzles with subsonic outflow. Then, the transition between the different classes of flow is investigated and ultimately a thermodynamic map of the reservoir conditions resulting in each functioning regime is produced. This map enables the identification of the thermodynamic region of interest for the observation of non-ideal nozzle flows.

To investigate supersonic flows past ramps, a general theory of oblique waves, i.e. waves generating a deflection of the supersonic stream, is developed. The analysis of oblique waves is performed by resorting to the concept of wave curves. The wave curve is the set of downstream states connected to a given upstream state by means of an oblique wave. Inspection of the wave curve structure for different upstream states reveals that, in addition to the conventional configuration consisting of compression oblique shocks and expansion Prandtl-Meyer fans, non-ideal configurations exist which include rarefaction shocks, compression fans and composite waves (combinations of shocks and fans). The connection between the upstream state and the configuration of the wave curve is explained and depicted through the use of thermodynamic maps. Further research in the context of oblique waves concerns oblique shocks featuring a non-ideal increase of the Mach number, which are systematically examined providing necessary conditions for their occurrence, and the extension of the typical shock angle–deflection angle diagram of oblique shocks with an additional half-plane corresponding to Prandtl-Meyer fans.

Then, the developed theory of oblique waves is applied to the investigation of shock reflections and interactions in non-ideal gasdynamics. The attention is placed on the neighbourhood of the singularity point where oblique waves intersect. Thanks to non-ideal effects such as the Mach number increase across shock waves, rarefaction shocks, compression fans and composite waves, the classical picture of shock reflections and interaction is enriched with new flow patterns. An overview of the most relevant configurations is given.

Finally, a realistic application where non-ideal supersonic flows occur is considered, namely the expansion in the turbine of an Organic Rankine Cycle (ORC) power plant. Focusing on supersonic stator vanes, firstly the flow is forced into a converging-diverging channel and then, at the trailing edge of each blade, oblique waves are generated due to the rotation imposed by the finite thickness of the trailing edge or to accommodate post-expansion or compression. The oblique waves generated on the pressure side of the blade propagate towards the suction side of the neighbouring blade and undergo a reflection when they impinge on the blade wall. The influence of non-ideal effects on the blade design and on the off-design performance is studied by means of numerical simulations. Advantages and disadvantages of operating in the non-ideal gasdynamic regime are discussed for a few specific examples, nonetheless providing more general considerations.